BAYESIAN NASH EQUILIBRIUM

complete information
- players
- actions
- payoffs

incomplete information
- players
  - types
  - common prior beliefs
- actions
- payoffs

Bayesian Nash Equilibrium (BNE)
- BNE is a set of strategies, one for each type of player, such that no type has incentive to change his or her strategy given the beliefs about the types and what the other types are doing.

- player 1 knows what type it is, but player 2 doesn't know what type player 1 is
  - one sided incomplete information

	RL	RR
UP	2.4, 3.2	0, 2.4
DOWN	1.8, .4	1, 1.2

← Best responses

FIND NASH EQUILIBRIUM
- * when 2 goes RL what is best response for 1? circle
- * when 2 goes RR what is best response for 1? circle
- * when 1 goes UP . . . for 2? line
- * when 1 goes down. . . for 2? line

FIND MS~~BN~~BNE:

		p RL	(1-p) RR
(p)	U	2.4, 3.2	0, 2.4
(1-p)	D	1.8, .4	1, 1.2

UP = 3.2(p) + .4(1-p)
+2.4(p) + 1.2(1-p)

RL = 2.4(p) + 0(1-p) + 1.8(~~1-~~p) + 1(1-p)

MS BNE:
UP = 1/2 RL = 5/8

2.4p + 1.8p + 1 - p
2.4p + .8p + 1
3.2p + 1 = 0

a type = p

	L	R
u	3, 4	1, 0
D	4, 3	2, 0

b type = 1-p

	L	R
u	6, 2	0, 4
D	5, 1	-1, 4

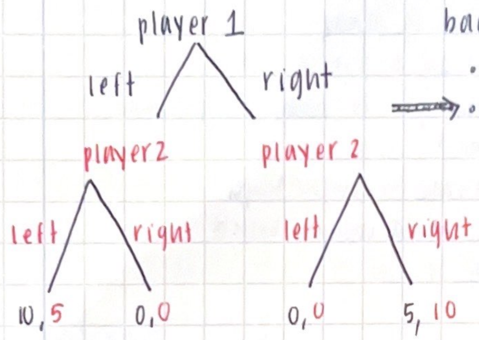
optimal strategy
- if P1 is type a they will choose Down
- if P1 is type b they will choose up
- if P2 is in world a then they choose ~~right~~ left
- if P2 is in world b then they choose right.
- *P2 doesn't know what world it's in but P1 knows what type it is and what its equilibrium is.

Based on type of P1 we can ~~pred~~ calculate
EU for P2:
u(left) = 3p + 2(1-p)
u(right) = 0p + 4(1-p)
3p + 2(1-p) > 0p + 4(1-p) → p > 2/5
P2 will play left if prior belief that P1 is in a ~~if pb~~ is > 2/5. otherwise ~~left~~ right.

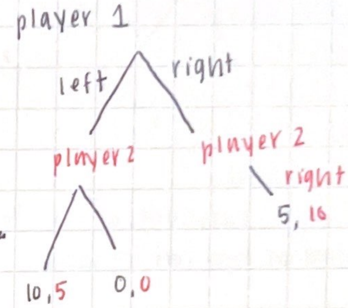
BNE:
- P1, a type chooses down
- P1, b type chooses up
- P2 chooses left if p > 2/5, right if p < 2/5 and mixes freely if p = 2/5.

* MSBNE we are calculating probability of player to make a move based on the payoffs of the ~~w~~ other player moving

SEQUENTIAL / DYNAMIC GAMES



backwards induction:
- player 2 goes left: chooses 10, 5
- player 2 goes right: chooses 5, 10



* the subgame perfect Nash equilibrium is (10, 5) *

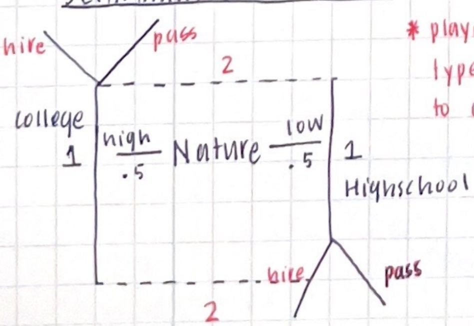
player 1 now chooses between going left or right. Right results in payoff of 5 but left has payoff of 10. Player 1 chooses left

* Neither player has incentive to change its strategy given the other players strategy

SIGNALING GAMES

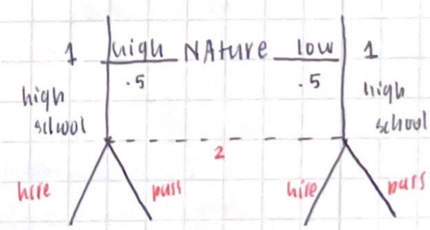
- Screening games: games where uninformed actors move first
- signaling games: games where informed actors move first.
  - informed actors might "signal" something to the uninformed actor.
- Types of Perfect Bayesian Equilibrium in signaling games: (PBE)
  - separating equilibrium
  - pooling equilibrium
  - semi-separating / partially pooling equilibrium } same thing

SEPARATING STRATEGIES:



(1)
* player 1 can infer if they are high type or low type from if they went to college or not.
* separating themselves based on strategies

POOLING STRATEGIES



* player 2 can't infer anything because 1 has pooled the strategies and adopted the same one.

* cournot reaction function: pg 6

* any monotonic transformation of a concave function is quasiconcave

① go through all concepts
② do practice problems
③ review papers

STAG-HUNT

	S	H
S	2,2	0,1
H	1,0	1,1

	S	H
S	a,a ← NE	c,b
H	b,c	d,d ← NE

- stag hunt differs from PD because there are two pure strategy NE. (~~cooperate and~~ both cooperate and both defect)

PRISONERS DILEMMA:

	C	B
C	a,a	c,b
B	b,c	d,d ← NE

- betrayal is always has a better payoff than cooperation
- strictly dominant strategy for A and B is to defect
- mutual cooperation yields a better outcome than mutual betrayal, but the choice to cooperate is not rational.
- thus, PD has a NE that is not Pareto Efficient.

PD example

P1 \ P2	C	D
C	1,1	3,0
D	0,3	2,2

MIXED STRATEGIES:

SHEILA \ THOMAS	C	D
A	10,16	14,24
B	15,20	6,12

dominant strategy? NO
pure strategy NE? YES, (B,C) and (A,D)
worse NE for thomas? (B,C)

- solve for mixed strategies:
- by principle of indifference, it must be that the expected payoff for sheila is the same whether she plays A or B

$$10(x) + 14(1-x) = 15(x) + 6(1-x)$$
$$x = 8/13$$

- For thomas we have:

$$16(y) + 20(1-y) = 24(y) + 12(1-y)$$
$$y = 1/2$$

- To verify that $x = 8/13$ and $y = 1/2$ is a mixed NE we calculate:

$A: 8/13(10) + 5/13(14) = 150/13$
$B: 8/13(15) + 5/13(6) = 150/13$
$C: 1/2(16) + 1/2(20) = 18$
$D: 1/2(24) + 1/2(12) = 18$

ZERO-SUM GAME

	C_1	C_2
C_1	-a,a	b,-b
C_2	c,-c	-d,d

- if one gains, another loses
- this means the result of a zero sum game is Pareto Efficient
- conflict game

zero sum example

cutter \ chooser	large	small
Even	(0,0)	(0,0)
UnEven	-10,10	10,-10

* cournot reaction functions: specify each firms ~~output~~ optimal output for each fixed output level of its opponent

* Every finite strategic form game has a mixed strategy equilibrium.

* we would like NE to be Pareto efficient *

- Debreu's theorem says that a quasiconcave function has a pure strategy Nash Equilibrium

- at mixed strategy NE, both players should have same expected payoffs from their two strategies.

- cournot: both players choose their actions simultaneously i.e try to simultaneously
- Stackelberg: player 1 chooses first, then player observes the output q1 and consequently chooses q2.
  - leader has the advantage

Extensive form games:

- the players payoffs are a function of previous moves
- if extensive form is finite, corresponding strategic form is finite and Nash theorem guarentees existence of a mixed strategy equilibrium.
- finite horizon games can be solved with backwards induction, while infinite horizon cannot.
- advantage is that it is clear what the order of play is. However, one player does not always observe the choice of another (strategic)

Repeated Games:

- introduces new equilibria: players may condition their actions on the way their opponents play in previous periods.
- in prisonners dilemma:
  - game played once: both defect (equilibrium)
  - game played finite number of times:
    - both defect is subgame perfect equilibrium
  - if horizon is infinite and $\delta > 1/2$:
    - cooperate until no player has defected. If any player defects, defect for the rest of the game
- In every subgame, no player can gain by deviating once from specified strategy and then conforming.
- repeated games can enforce cooperation

FOLK THEOREM:

- if players are sufficiently patient, then any feasible, individually rational payoffs can be enforced as an equilibrium
- in the limit of extreme patience, repeated play allows virtually any payoff to be an equilibrium outcome
- when players are patient, any finite one period gain from deviation is outweighed by even a small loss in utility in every future period.

MIXED STRATEGY
(BAYESIAN NE)

	Enter	Dont
Build	0, -1	2, 0
Dont	2, 1	3, 0

Build cost High (p1)

	Enter	Dont
Build	3, -1	5, 0
Dont	2, 1	3, 0

Build cost Low (1 - p1)

P1: Build?

P2: enter?

P1 knows its type

P2 does not know type for P1

If cost is HIGH: dom strategy for P1 is dont build

If cost is LOW: dom strategy for P1 is build

strategy for player 2 to enter:

$$1 \cdot p1 + (-1)(1-p1) > 0$$

$$p1 > 1/2$$

	Enter	Dont
Enter	0, -1	2, 0
Dont	2, 1	3, 0

High (P1)

	Enter	Dont
Enter	1.5, -1	3.5, 0
Dont	2, 1	3, 0

LOW (1-P1)

PAGE 280 (MS BNE)

BNE

Happy (0.5): Alice (rows), Bob (columns)

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

UNHAPPY (0.5): Alice (rows), Bob (columns)

	B	F
B	2, 0	0, 2
F	0, 1	1, 0

HAPPY Alice chooses B → Bob BR: B (1)

Alice chooses F → Bob BR: F (2)

- games of perfect information - sequential games
- games of imperfect information - simultaneous games

(1) Alice chooses B

Alice $EU(B) = 2(0.5) + 0(0.5) = 1$

Alice choose F:

Alice $EU(F) = 1(0.5) + 0(0.5) = 0.5$

Best response is for Alice to choose B. The BNE is (B, B if happy and F if unhappy)

UNHAPPY Alice chooses B → Bob BR: F (3)

Alice chooses F → Bob BR: B (4)

FOLK THM CNTND: repeated play with patient players not only makes cooperation possible (more efficient payoffs) but it leads to a large set of other equilibrium outcomes.

TIMING

INFORMATION	simultaneous	sequential
comp.	Nash	SPE
incomp.	Bayesian Nash	Perfect Bayesian

signaling?

- everything before was STATIC!
- for static games of incomplete information:
  - game/payoff depends on type of players
  - player knows its own type, but not the type of others.

* • DYNAMIC GAMES:
- players have the chance of updating their beliefs based on the observed actions of the other players.
- backward induction was used in games with perfect information
- PBE: strategies are optimal given the beliefs, and the beliefs are obtained from equilibrium strategies and observed actions
  - no backward induction! (SPE)

PAGE 24: SIGNALING GAMES

FICTICIOUS PLAY:
- repeated game
- each players beliefs of opponents ~~are~~ strategy are updated by looking at what happened
- player then plays best response according to his belief
- players observe only their own matches and play a best response to the historical frequency of play.
- ficticious play process converges for 2 person zero sum game

EXTERNAL REGRET: difference between the payoffs achieved by the strategies prescribed by the given algorithm and the payoffs achieved by ~~the~~ any other fixed sequence of decisions.

[compares the performance of an online algorithm, selecting among N actions, to the performance of the best of those actions in hindsight]
- need to know payoff that would be obtained for all possible strategies.
  - no internal regret learning converges to the correlated Nash equilibrium

INTERNAL REGRET.
- compares the loss of an online algorithm to the loss of a modified online algorithm, which consistently replaces one action by another.
  - converges to correlated NE
- regret of a player at time t: difference between the payoffs achieved using its strategy of choice (i) and the payoffs that could have been achieved had strategy $j \neq i$ been played instead.

SIGNALING GAME EXAMPLE

A2

Weak (1/2):

A1	occupy	retreat
occupy	*x, x	10, 0
retreat	0, 10	0, 0

Strong (1/2):

A1	occupy	retreat
occupy	x, x	0, 10
retreat	0, 10	0, 0

A1 has dom strategy to retreat

1) BNE if $x = -20$? (for Army 1)(to attack)

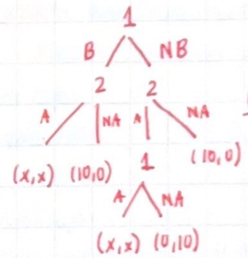
$-20(1/2) + 10(1/2) = -5 > 0$

2) BNE if $x = -3$

~~+ 2/p~~ $-3(1/2) + 10(1/2) = 3.5 > 0$

3) Model as Bayesian signaling game:

(A1 can signal type by burning bridge)



P1 \ P2	A	NA
B	(x, x)	(10, 0)
NB	$\frac{x}{2}, \frac{x+10}{2}$	(10, 0)

4) for what value of x does A1 have to burn bridge?
- look at Burn row. If they Burn and attack, this is the same as occupy and occupy. so $x + 10 > 0$ from previous matrix and $x + 0 > 0$.

* under ficticious play, a player chooses a best reply to her belief, whereas under no regret dynamics, she chooses a better reply *

* in a repeated game, a player has a regret for an action if she could have obtained a greater average payoff had she played that action more often in the past.

* dynamic = sequential / repeated
* static = simultaneous

CORRELATED EQUILIBRIUM:

- A correlated strategy is called a correlated equilibrium if it is better off for every player to obey her recommended ~~recommendations~~ strategy if she believes that all other players obey their recommended strategies.
- the difference between MSNE and CE is that mixing is independent in NE. With more than two players, it may be important in CE that one player believes others are correlating their strategies.
  - ↳ if a game is repeated infinitely many times s.t every player plays according to a certain regret minimization strategy → the empirical frequencies of play converge to the set of correlated equilibria

SUPERMODULAR GAMES:

- games in which each players marginal utility of increasing its strategy rises with increases in its rivals strategies
- have a pure strategy NE

POTENTIAL GAMES

- a game is a potential game if the incentive of all players to change their strategy can be expressed using a single global function called a potential function.

PARETO EFFICIENT EXAMPLE:

①

	a_1	a_2
a_1	$\boxed{2,3}$	$\boxed{-2,7}$
a_2	$\boxed{6,-5}$	6, -1

②

	a_1	a_2
a_1	2, 3	$\boxed{-2,7}$
a_2	$\boxed{6,-5}$	$\boxed{3,5}$

- circles are NE and squares are PE.
- in ① a_2, a_2 can be improved for both players by going to a_1, a_1
- in ② a_1, a_1 could be made better by going to a_2, a_2 so it is not PE.

SUBGAME:

- starting at any decision point in the game, a player strategy (from that point on) is a best response to the strategies of other players

COORDINATION GAME CONTINUED:

	L	R
U	2, 4	1, 3
D	1, 3	2, 4

- a players optimal move depends on what he expects the other player to do.
- they both do better if they coordinated than if they played an off equilibrium combination of actions.
- simultaneous game
- a player will earn a higher payoff when they select the same course of action as another player
- multiple NE in which players choose matching strategies

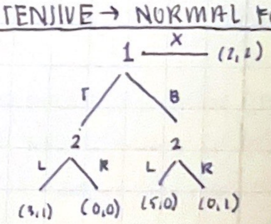
correlated Equilibrium

Game of chicken ☺

	D	C
D	0, 0	7, 2
C	2, 7	6, 6

- Pure strategy NE: (D,C) / (C,D) and MSNE where both players chicken out w/ prob 2/3
- 3rd party draws 3 cards: (C,C), (D,C) and (C,D) with same probability (1/3)
- 3rd party informs strategy to players, but players dont know others strategies.
- suppose P1 has C and P2 has C/D with prob 1/2. Expected utility of D is: $7(1/2) + 0(1/2) = 3.5$ and EU of chicken is: $2(1/2) + 6(1/2) = 4$. player wants to stay with chicken. If P1 has D then P2 must have C and P1 will stay w/ D.
- Neither player has incentive to deviate.
- CE: $7(1/3) + 2(1/3) + 6(1/3) = 5 > $ MSNE

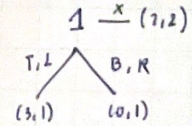
EXTENSIVE → NORMAL FORM



Normal Form:

	LL	LR	RL	RR
X	2,2	2,2	2,2	2,2
T	3,1	3,1	0,0	0,0
B	5,0	0,1	5,0	0,1

NE: (T, LR), (X, RR)



P1 P2
↓ ↓
SPE: (T, LR)

- pareto efficient: impossible to make one party better w/o making someone worse
- pareto improvement: occurs when at least one individual becomes

- incomplete information means there are things you simply dont know such as opponents strategies or payoffs
- imperfect information means you won't know when or if an opponent makes a move

For static games w/ incomplete information:

- game / payoffs depend on the type of players.
- A player knows its own type but it does not know the types of other players
- transform game of incomplete information → game of imperfect information:
  - assign probabilities to type of players
  - perceived as a move by nature
  - represents players apriori belief on the types of other players.

* more on pg 69.

OTHER INFO:

- cournot adjustment: unique NE is at the intersection of the reaction curves.
  - the process converges to NE from any starting point (globally stable)
- fictitious play: players choose their actions in each period such that they can maximize their expected payoff, with respect to their belief for the current period.
- Fictitious converges for zero-sum, identical interest, potential, nonzero-sum (2 player with max 2 strategies)

* non cooperative game examples:

- zero sum
- rock paper scissors
- prisoners dilemma

* NE vs. ~~Stackelberg~~ SPE

- in NE every party is playing a best reply on the equilibrium path.
- In SPE every party is also planning to play a best reply off the equilibrium path

PAPERS

① A new Framework for Power Control in Wireless data networks: Games, utility and Pricing:

- game is how to effectively manage radio resources for users of a wireless data network.
- players: individual users that adjust their transmitter power in order to maximize utility
- actions: adjust individual power levels
- utility: measure of satisfaction that user gets from accessing network.
- users signal acts as an interference to the other users signals
- tradeoff between power and achieved SIR

UTILITY FUNCTION PROPERTIES:

- monotonically increasing function of users SIR
- monotonically decreasing function of users transmitter powers

PRICING:

- increase in power leads to increase in SIR and more interference to users nearby
- bring a pareto improvement in utilities of users.

② A STACKELBERG GAME APPROACH (dependable demand response)

- players: utility companies and users
- UC's act first, then users decide based on prices
- utility function increases with the amount of electricity that the user can consume
- UC's play noncooperative price selection game to find optimal unit price. users calculate best response
- unit price is at NE. For users, equilibrium strategy is any strategy that leads to optimal response

③ BAYESIAN GAME APPROACH FOR INTRUSION DETECTION IN AD HOC NETWORK

- model interactions between pairs of attacking / defending nodes within the network
- game is incomplete information game where defender is uncertain of opponent (regular / malicious)
- defender selects strategies based on belief of type of opponent
- static { advantage of static game is less power to monitor because of efficient monitoring system
- dynamic { infinite horizon & mixed strategies depend on history of game
- for static: does not take into account game evolution and defender has fixed prior beliefs about opponent type

STATIC:

- mixed BNE when D's belief of i is malicious in high
- pure BNE when D's ... is low

DYNAMIC:

mixed strategy PBE

- dynamic game is more realistic because defender can update beliefs dynamically based on new observations & can adjust monitoring strategy accordingly

ADAPTIVE CHANNEL ALLOCATION SPECTRUM ETIQUETTE FOR COGNITIVE RADIO NETWORKS

(4) • cognitive radio can learn from history and adjust according to current state of environment

• formulated as POTENTIAL game and NO REGRET GAME

POTENTIAL GAME

• converges to NE point for cooperative
• use U2. cannot use U1 because it lacks necessary symmetry

FRAMEWORK

• channel allocation problem modelled as normal form game
• players: cognitive radios
• actions: select new transmission parameters and transmission frequencies
  • actions influence own performance as well as performance of neighboring players
    • decisions are based on perceived utility associated with each possible action
• utility functions characterize a users level of cooperation and support a selfish and cooperative spectrum sharing etiquette

• in terms of SIR / achievable throughput, average performance for both types of games are similar
• potential game has best performance, but is limited to cooperative environment
• no regret is applicable to both and requires minimal amount of information.

FICTICIOUS PLAY CONTINUED:

• a learning rule in which each player presumes that the opponents are playing stationary (possibly mixed strategies). At each round, each player thus best responds to the empirical frequency of play of their opponent.
• Flawed if opponents strategy is non stationary.

NO REGRET GAME

• converges to pure strategy NE for cooperative and mixed ~~for~~ equilibrium for selfish users
• cooperative (U2) selfish is (U1)
• for selfish users, amount of info is minimal
  • users need to measure interference at recievers (U1) and update weights for channel selection to favor channel with minimum interference
• for cooperative users, info to compute U2 is similar to potential game

UTILITY FUNCTIONS

• U1 represents selfish user
• selfish user values a channel based on that level of interference on that channel
• needs only interference measurement of a particular user on different channels
• U2 represents cooperative user
• accounts for interference seen by a user on a channel as well as interference this choice will have on neighboring nodes
• more info as it has to probe packets in order to measure interference caused to neighbors

OTHER NOTES:

• cournot equilibrium - rival firms produce a homogenous product and each attempts to maximize profits by choosing how much to produce. All firms choose output simultaneously. Resulting equilibrium is a NE in quantities called a Cournot equilibrium

type of learning
• cournot adjustment - each firm assumes that the ~~next~~ rivals output will not change from one period to the next. therefore, each firm changes its output sequentially. Adjustment process converges to Cournot Equilibrium.